Project objectives

1. Elaboration and development of integral and hybrid methods for solving the electromagnetic field problems within structures with moving parts and non-linear media.

1.1. The hybrid methods FEM-BEM. A FEM mesh is created only in the ferromagnetic bodies and those were eddy currents are induced; the boundary condition is a mix one and it is given by the rigidity matrix that results from Boundary Elements Method (BEM). The movement is reflected only on the boundary condition. For 2D structures the integral equation of vector potential on the boundary is:

$\alpha A(\boldsymbol{r}) = -\oint_{\partial\Omega} \frac{(\boldsymbol{R}\cdot\boldsymbol{n'})}{R^2} A(\boldsymbol{r'})dl' + \oint_{\partial\Omega} \ln\frac{1}{R}\frac{\partial A(\boldsymbol{r'})}{\partial n'}dl' + A_0$ ....(3), where *A* is the vector potential; $\partial\Omega$ is the boundary of the ferromagnetic bodies; α is the solid angle that a small vicinity of $\partial\Omega$ can be seen from the observation point; *r*, *r'* are the position vectors of the observation point and source point, respectively; *R* = *r* - *r'*; *n'* is the unit normal external vector; A_0 is the vector potential given by external field sources. On ferromagnetic bodies we write $A = \sum_{i=1}^{N} \alpha_i \varphi_i$, where φ_k are test functions (for ex. nodal elements), and *N* is the number of these functions. The weak form of the equation for vector potential is: $-\oint_{\partial\Omega} \varphi_k \nu \frac{\partial A}{\partial n} dS + \int_{\Omega} grad\varphi_k \cdot \nu gradA d\Omega$ $= \int_{\Omega} \varphi_k J d\Omega$, *k*=1,2,...,*N* ....(3'). Between the tangential components of the magnetic field strength (derivatives of potential on the normal direction) from inner and outer domain there is the following relation: $\frac{\partial A}{\partial n'} = -\frac{1}{\mu_r}\frac{\partial A}{\partial n}$ ...(4). The boundary is approximated with a polygonal line; on this boundary the variation of the potential is linear and $\nu\frac{\partial A}{\partial n}$ is constant. The numerical form of the integral equation is: $Z(rot\boldsymbol{A})_t + W\boldsymbol{A}_t = \boldsymbol{A}_0$ ...(5) that, together with the conditions and equation (3) form the equation system from FEM-BEM hybrid procedure.

The credibility of the methods results from the fact that a part of the results have been already published or presented within scientific conferences /6/, /7/.

Comparing to FEM, some of the great advantages of the procedure presented above are: structures with moving bodies can be considered; there are not parasite forces in the air (where the equations of the magnetic field are exactly verified); it is not necessary to introduce an artificial boundary; the mesh is created only on the ferromagnetic domains.

The matrix of the equations system has some disadvantages: great dimensions, it loses from the property of rare matrix specific to FEM methods (the lines associated to the boundaries are full) and it is not symmetrical. We propose an iterative method for solving the equations system:

- admitting as known the value $\frac{\partial A}{\partial n'}$ on the boundary of the outer domain, the value of *A* is determined using the relation (5): $A = W^{-1}Z\frac{\partial A}{\partial n'} + W^{-1}A_0$ ...(6)

- the field problem on the shield is solved, having boundary Dirichlet conditions. The matrix of the system is well conditioned, being rare, symmetrical and diagonal dominant. Rare matrix techniques can be used.

- after solving the filed problem inside the shield, we obtain the values $\frac{\partial A}{\partial n}$ on the boundary. The values $\frac{\partial A}{\partial n'}$ are corrected for the outer problem using the relation (4).

In the 3D case, we propose a new integral equation:

$\alpha \boldsymbol{A}(\boldsymbol{r}) = -\oint_{\partial\Omega} \frac{\boldsymbol{n}'}{R} \times (\nabla' \times \boldsymbol{A}(\boldsymbol{r}'))dS' + \oint_{\partial\Omega} \frac{\boldsymbol{R}}{R^3} \times (\boldsymbol{n}' \times \boldsymbol{A}(\boldsymbol{r}'))dS' + \boldsymbol{A}_0$, that represent the boundary condition for the interior FEM procedure. Edge elements and the condition for topological calibration are used.

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1.2. *The integral equation for eddy currents.* We propose the use of the local reference systems that are attached to the moving parts, *where the local form for law of electromagnetic induction within immobile media is valid and we can write:* $\int_0^t \boldsymbol{E} d\tau = -\boldsymbol{A} + gradV$. Outside the conductive bodies domains, the stationary state is valid and the B-S-L formulae can be applied. In 2D cases, where we have bodies with polarizations, we propose a new form of the integral equations for eddy currents:

$\rho \underset{\bullet}{J}(\boldsymbol{r}) + \frac{\mu_0}{2\pi}\int_\Omega J(\boldsymbol{r}')\ln\frac{1}{R}dS' = -A_0 - \frac{\mu_0}{2\pi}\int_\Omega \boldsymbol{k}\cdot(\nabla \times \boldsymbol{M}(\boldsymbol{r}'))\ln\frac{1}{R}dS' + C_l$, where A_0 is the vector potential given by the external field sources and $\boldsymbol{M}$ is the magnetization that can come from the polarization method, when we have non-linear ferromagnetic media, and $\underset{\bullet}{\boldsymbol{J}} = \int_0^t \boldsymbol{J}(\tau)\mathbf{d}\tau$. The nucleus of the integral equations $\ln\frac{1}{R}$ can be modified in time. Volume elements are used (or surface elements, in the 2D models).

In 3D structures, we propose the equation:

$\rho \underset{\bullet}{\boldsymbol{J}} + \frac{\mu_0}{4\pi}\int_{\Omega_C}\frac{\boldsymbol{J}}{r}dv + gradV = -\frac{\mu_0}{4\pi}\int_{\Omega_0}\frac{\boldsymbol{J}_0}{r}dv - \frac{\mu_0}{4\pi}\int_{\Omega_F}\frac{\boldsymbol{M}\times\boldsymbol{r}}{r^3}dv$, where the potential V is unknown. We write $\underset{\bullet}{\boldsymbol{J}} = rot\boldsymbol{T}$, cu $\boldsymbol{T} = \sum_{j=1}^{n}\alpha_j(t)\boldsymbol{N}_j$, where $\boldsymbol{T}$ are the edge elements of co-tree, in the edges graph and the integral equation is projected on the functions $rot\boldsymbol{N}_k$, eliminating, in this way, the potential V.

1.3. *Approaching the non-liniarities.* The ferromagnetic media, with the constitutive relation $\boldsymbol{H} = \mathbf{F}(\boldsymbol{B})$ are replaced by linear media with the constitutive relation $\boldsymbol{B} = \mu(\boldsymbol{H} + \boldsymbol{M})$. The magnetizations $\boldsymbol{M}$ are iteratively corrected as a function of the magnetic flux $\boldsymbol{B}$. For the computational medium the magnetic permeability $\mu = \mu_0$ can be chosen. The medium being linear and homogenous from the magnetic point of view, solving an electromagnetic field problem can be done, within one iteration, through the solution of the integral equation of the current density. The reference systems of the moving bodies are used. The magnetization M appears in the free term of the integral equation. The nuclei of these integral equations are time-functions.

2. The computation of the forces

The magnetic force is calculated using the formula: $\boldsymbol{F} = \frac{1}{\mu_0}\oint_\Sigma\left[(\boldsymbol{nB})\boldsymbol{B} - \boldsymbol{n}\frac{B^2}{2}\right]dA$, where the surface of integration Σ surrounds the referring body. We denote $\boldsymbol{J}_k$ the uniform current density from the conductive sub-domain k, k=1,...,n_C and $\boldsymbol{M}_k$ the uniform magnetization from the polyhedral ferromagnetic sub-domain ω_k, k=1,...,n_F. We ca apply B-S-L formulae and we obtain **the new relations:**

$$\boldsymbol{F} = \frac{\mu_0}{16\pi^2}\sum_{j=1}^{n_j}\int_{\sigma_j}\left[\left(\boldsymbol{n}_j\sum_{k=1}^{n_C}\boldsymbol{J}_k\times\boldsymbol{A}_{kj} + \boldsymbol{n}_j\sum_{k=1}^{n_F}\boldsymbol{M}_k\cdot\overline{\overline{\boldsymbol{C}}}_{kj} + \boldsymbol{n}_j\boldsymbol{D}_j\right)\times\left(\sum_{k=1}^{n_C}\boldsymbol{J}_k\times\boldsymbol{A}_{kj} + \sum_{k=1}^{n_F}\boldsymbol{M}_k\cdot\overline{\overline{\boldsymbol{C}}} + \boldsymbol{D}_j\right) - \frac{\boldsymbol{n}_j}{2}\left(\sum_{k=1}^{n_C}\boldsymbol{J}_k\times\boldsymbol{A}_{kj} + \sum_{k=1}^{n_F}\boldsymbol{M}_k\cdot\overline{\overline{\boldsymbol{C}}}_{kj} + \boldsymbol{D}_j\right)^2\right]dA_j,$$

where $\boldsymbol{A}_{kj} = \oint_{\partial\omega_k} \frac{\boldsymbol{n}_k}{r} dA$; $\overset{=}{\boldsymbol{C}}_{kj} = \oint_{\partial\omega_k} \frac{(\boldsymbol{R};\boldsymbol{n}) - (\boldsymbol{R}\cdot\boldsymbol{n})}{r^3} dA$; $\boldsymbol{D}_j = \sum_{k=1}^{n} i_k \oint_{\Gamma_k} \frac{\boldsymbol{dl}\times\boldsymbol{R}}{R^3}$. For moving bodies, a part of the terms $\boldsymbol{A}_{kj}$,$\boldsymbol{C}_{kj}$ *si* $\boldsymbol{D}_j$ must be recalculated, namely those terms that refer to the surfaces *S* and conductive or ferromagnetic bodies that moves one towards others. For electrical forces, such as those forces that appear in the trajectories of the electron fascicles or in the electrical separators, the formulae of the local densities of the forces can be used.

3. Solving the equation of dynamic equilibrium

The electromagnetic force is deduced if we solve a more complicated problem of quasi-stationary electromagnetic field. The solution of the field problem depends on the velocity and position of the body. Almost every time more degrees of freedom must be taken into consideration. The problem is, thus, non-linear and a numerical algorithm for solving it implies the insurance of the stability conditions of the procedure.

Further on we present a modality of approaching this objective. Let us admit a simple case with one degree of freedom and without mechanical frictions. The equation of dynamic equilibrium is: $m\frac{dv}{dt} = F(x,v)$ ... (7), $v = \frac{dx}{dt}$. An iterative algorithm is proposed. We admit that within the time interval [*t*,*t*+*Δt*] the movement is an uniform accelerate one. The numerical form of the above equations is: $m\frac{v^{(p+1)}(t+\Delta t) - v(t)}{\Delta t} = \tilde{F}^{(p)}$, and thus: $v^{(p+1)}(t+\frac{\Delta t}{2}) = \frac{v^{(p+1)}(t+\Delta t) + v(t)}{2}$.

Then $x^{(p+1)}(t+\Delta t) = x(t) + v^{(p+1)}(t+\frac{\Delta t}{2})\Delta t$ and $x^{(p+1)}(t+\frac{\Delta t}{2}) = x(t) + \frac{v^{(p+1)}(t+\Delta t) + 3v(t)}{8}\Delta t$. Considering the fact that the conductive domanins where divided in *n* tetrahedral sub-domains ω_k where the current density $\boldsymbol{J}_k$ is supposed to be constant, we have: $\boldsymbol{B} = \sum_{k=1}^{n} \boldsymbol{J}_k \times \boldsymbol{D}_k$, where $\boldsymbol{D}_k = \frac{\mu_0}{4\pi}\int_{\omega_k} \frac{\boldsymbol{r}}{r^3} dv$ or $\boldsymbol{D}_k = \frac{\mu_0}{4\pi}\oint_{\partial\omega_k} \frac{\boldsymbol{n}}{r} dA$. We take the average value of the force on the time interval [*t*,*t*+*Δt*]: $\tilde{\boldsymbol{F}} = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} \boldsymbol{F} dt$. Admitting that, within the time interval [*t*,*t*+*Δt*] the movement is an uniform accelerated one and known at the iteration *(p)*, the current density can be calculated with the hypothesis of an imposed trajectory. The time-step used to determine the current density is different from that one used in the equation of dynamic equilibrium. A Crank-Nicholson procedure can be used.

4. The stability of the trajectory

If we want to study the global stability with respect to the normal trajectory, then we have to produce a perturbation at any moment, that mean to add a force that acts for a short time to the right part of the equation (7) and to find that the new trajectory tends asymptotical to the normal one (Fig. 1).

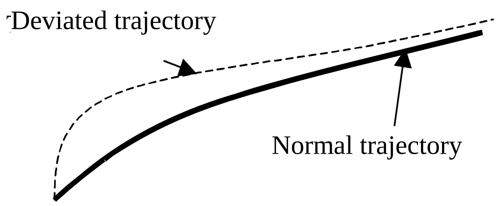

Fig. 1 For stability of the trajectory

A lot of complex problems regarding trajectories should be solved. A substantial simplification of the analysis of trajectory stability can be done, analyzing the local stability. At one point of the trajectory, supposed to be known, an arbitrary small deviation from the trajectory can be done and, supposing that we have the same velocity, the electromagnetic force is determined. If this force tends to bring back the body on the normal trajectory then we say that we have local stability. For instance, for the electromagnetic force we have the relation:

$\delta\boldsymbol{F} = \frac{1}{\mu_0} \oint_{\partial D_0} \left(\boldsymbol{B}(\delta\boldsymbol{Bn}) + \delta\boldsymbol{B}(\boldsymbol{Bn}) - \delta\boldsymbol{BB}\right) dA$, where $\boldsymbol{B}$ is calculated at the point of the normal trajectory and

$\delta\boldsymbol{B} = \frac{\mu_0}{4\pi}\left[\sum_{k=1}^{N_c}\left(\int_{D_k} \frac{\delta\boldsymbol{J}\times\boldsymbol{r}}{r^3} dv\right) + \sum_{k=1}^{N_c}\left(\int_{D_k} \boldsymbol{J}\times\left(\frac{\delta\boldsymbol{d}}{r^3} - 3\frac{\boldsymbol{r}(\boldsymbol{r}\delta\boldsymbol{d})}{r^5}\right) dv\right)\right]$, where $\delta\boldsymbol{d}$ is the displacement from the point of the trajectory. The integral equation of the current density difference is:

$$\rho\delta \underset{\bullet}{\boldsymbol{J}} + \frac{\mu_0}{4\pi}\left[\sum_{k=1}^{N_c}\left(\int_{D_k} \frac{\delta\boldsymbol{J}}{r} dv\right) + \sum_{k=1}^{N_c}\left(\int_{D_k} \frac{(\delta\boldsymbol{d}\times\boldsymbol{J})\times\boldsymbol{r}}{r^3} dv\right)\right] + gradV = -\frac{\mu_0}{4\pi}\int_{\Omega_0} \frac{(\delta\boldsymbol{d}\times\boldsymbol{J}_0)\times\boldsymbol{r}}{r^3}\mathbf{d}v$$

If the small deviation from the normal trajectory is made on the direction $\boldsymbol{u}$, then $\delta\boldsymbol{d}=\boldsymbol{u}\delta s$. In the relations from above the small variation are replaced by the derivatives with respect to s. $\frac{\delta\boldsymbol{F}}{\delta s} = \frac{1}{\mu_0}\oint_{\partial D_0}\left(\boldsymbol{B}(\frac{\delta\boldsymbol{B}}{\delta s}\boldsymbol{n}) + \frac{\delta\boldsymbol{B}}{\delta s}(\boldsymbol{Bn}) - \frac{\delta\boldsymbol{B}}{\delta s}\boldsymbol{B}\right) dA$,

$\frac{\delta\boldsymbol{B}}{\delta s} = \frac{\mu_0}{4\pi}\left[\sum_{k=1}^{N_c}\left(\int_{D_k} \frac{\left(\frac{\delta\boldsymbol{J}}{\delta s}\right)\times\boldsymbol{r}}{r^3} dv\right) + \sum_{k=1}^{N_c}\left(\int_{D_k} \boldsymbol{J}\times\left(\frac{\boldsymbol{u}}{r^3} - 3\frac{\boldsymbol{r}(\boldsymbol{ru})}{r^5}\right) dv\right)\right]$. The equation of the derivative with respect to s of the integral current density is: $\rho\underset{\bullet}{\left(\frac{\delta\boldsymbol{J}}{\delta s}\right)} + \frac{\mu_0}{4\pi}\left[\sum_{k=1}^{N_c}\left(\int_{D_k} \frac{\left(\frac{\delta\boldsymbol{J}}{\delta s}\right)}{r} dv\right) + \sum_{k=1}^{N_c}\left(\int_{D_k} \frac{(\boldsymbol{u}\times\boldsymbol{J})\times\boldsymbol{r}}{r^3} dv\right)\right] + gradV =$ $-\frac{\mu_0}{4\pi}\int_{\Omega_0} \frac{(\boldsymbol{u}\times\boldsymbol{J}_0)\times\boldsymbol{r}}{r^3}\mathbf{d}v$. The settlement of the above equation is done on the small time interval $[t,t+\sigma]$. For simplicity, we supposed that the media is linear, so the magnetization $\boldsymbol{M}$ is missing.

Methodology of the research

The project will be focused on five directions of fundamental research: 1) Elaboration and development of integral and hybrid methods to solve the electromagnetic field problem. Structures with moving parts and non-linear media will be also considered. Stationary problems of electric and magnetic field and eddy currents problems will be solved. Formulae and procedures for electromagnetic field computation in the air, where the field is continuous and undefined derivable, will be established.

Choosing simple structures, where the analytical solution can be found, the obtained solutions can be verified. 2) Methods of electromagnetic forces computation using the results of integral and hybrid equations. The flux of Maxwell tensor is used when we determine the magnetic forces exerted on the moving parts or the local forces formulae are used when we determine the trajectories of the electrical charged particles. 3) Methods for numerical solution of the equations of dynamic equilibrium. Using integral or hybrid methods, the correction of the electromagnetic force is made without re-building the mesh 4) Methods of numerical analysis of the trajectories stability. 5) Algorithms and computation programs. The research team includes 3 professors with great experience within Electromagnetism, 2 young researchers that have recently sustained their doctoral thesis, and a PhD student. As a consequence, three working teams with two members will be formed, each one coordinated by a professor. Within the activity of these teams other young PhD students will be co-opted, but the responsibility of the research activity belongs to the leaders of the teams. The ordinary and extraordinary research reports will be analyzed in the presence of the whole 6 people’s staff, the connection between the activities of the team being the project manager responsibility. The results will be **reported in ISI quoted magazines (at least 6 articles)** and they will be communicated within scientific conferences with reviewers.